PenLab: Towards Understanding of Active Collaboration for Solid Geometry Teaching

Dehui Kong
kindhui62@gmail.com
School of Information Science and Engineering, University of Jinan
Jinan, China

Hongyue Wang
hongyue@exertiongameslab.org
Exertion Games Lab, Department of Human-Centred Computing, Monash University
Melbourne, Victoria, Australia

Sijie Zhou
koaikoo323@gmail.com
Independent Researcher
Nanjing, China

Hong Cui
hcui1018@gmail.com
School of Information Science and Engineering, University of Jinan
Jinan, China

Zhiquan Feng*
ise_fengzq@ujn.edu.cn
School of Information Science and Engineering, University of Jinan
Jinan, China

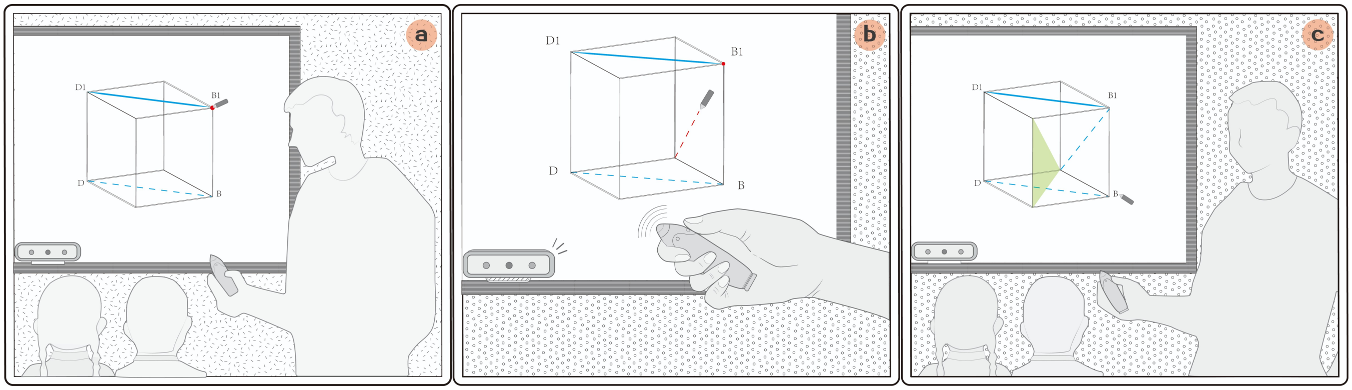

Figure 1: PenLab: (a) Teacher employing the smart pen for solid geometry instruction; (b) System comprehending teacher intentions, actively guiding the teacher through guiding lines for point-line interaction; (c) Teacher successfully completing point-line interaction.

ABSTRACT

With the continuous advancement of technologies such as VR and sensors, pen-based interaction has transcended the limitations of 2D interfaces. Although research on aspects of Human-Pen Interaction, such as pen grip, gesture operations, and tactile support, has been extensive, a thorough exploration of active collaborative interaction with the pen remains relatively limited. Active collaboration in Human-Pen Interaction refers to the system understanding the participants' interaction intentions and actively providing feedback and guidance for collaboration. Facing the dilemma of inaccurate selection in pen interactions for teachers in solid geometry teaching, we have designed an interactive system for solid geometry teaching with active collaboration capabilities, consisting of a depth camera, a smart pen embedded with multiple sensors, and a virtual geometry teaching platform. By inviting participants to experience the system and collecting quantitative data on user experience and attitudes, the results indicate that the system can assist in geometry teaching with more precise and flexible interaction methods.

CCS CONCEPTS

- **Human-centered computing → Interactive systems and tools.**

KEYWORDS

smart pen, human-pen interaction, virtual solid geometry teaching, active collaborative interaction

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1 INTRODUCTION

Imagine the scenario when you first encountered solid geometry in a math class: transitioning from familiar 2D shapes to intricate 3D figures, exploring rotations, flips, and constructing auxiliary lines for geometric proofs. Due to the abstract nature of solid geometry and the complexity of 3D space, students find it challenging to accurately visualize spatial figures in their minds, leading teachers to invest significant class time in explaining solid geometry concepts. However, traditional geometry teaching still faces issues of unclear explanations of geometric knowledge and difficulties in drawing geometric shapes. To address these challenges, "Dynamic Mathematics" [22] has become a prevalent teaching approach. Visualization-assisted teaching software such as GeoGebra [22], Cabri3D [16], and technologies supporting geometry teaching through VR and AR like VRMath [26], Construct3D [11], boast powerful 3D display capabilities, effectively enhance the monotony and abstractness issues in traditional geometry teaching [9]. Additionally, natural gesture-based interaction methods like HandWaver [5] construct geometric shapes through a series of gestures and visualize dynamic changes in geometric shapes using virtual reality technology. These technologies enable teachers to present geometric instructional content more seamlessly.

Although many teaching assistance techniques available now have the aforementioned potential, we find that there still exist (1) software learning barriers [23], (2) low usage frequency [23], (3) some tools are only suitable for individual use and not applicable to secondary school classroom teaching [24], (4) a relatively single input mode and weak understanding of intentions, lack of collaborative interaction capability, only completing the teacher's instructions passively, and unable to actively interact[17], and other issues. Specifically, in the context of solid geometry teaching, handheld controllers [14] mainly rely on wrist rotation for target selection. However, when it comes to very precise point-line selection, pen-based interaction can more fully leverage the dexterity of human fingers, achieving higher precision in pointing and selection [12]. In addition, teachers expect future interactive tools can assist in teaching, especially in solid geometry teaching, in a more precise and flexible manner [3].

Therefore, we propose PenLab (Figure 1), an interactive system designed for solid geometry teaching. PenLab consists of a depth camera, a smart pen embedded with multiple sensors, and a virtual geometry teaching platform, aiming to provide technical support for teachers in explaining solid geometry. We validated and evaluated this interactive system through a teaching experiment on the platform focusing on "solving the volume of a triangular pyramid."

Specifically, our contributions can be summarized as follows:

(1) Presenting a PenLab designed for solid geometry teaching, the system can capture and understand the teacher's actions in real-time, providing instant feedback and interaction.

(2) Presenting an active collaborative point-line interaction strategy, the strategy allows the system to understand the teacher's point-line selection intentions and actively provide feedback and guidance, engaging in active collaboration to assist the teacher in precise point-line interactions.

2 RELATED WORK

2.1 Pen-based Interaction

In the field of human-computer interaction, pen-based interaction, as a natural and intuitive input method, has expanded the ways in which participants interact with computers.

During the interaction process in 2D spatial environments, researchers have explored parameters of the pen such as scrolling [1], tilting [20], pressure [25], as well as combinations of grip and motion [27], altering the interface input mode. Hinckley et al. [8] investigated the possibilities of combining pen interaction with multi-touch, creating a novel pen-tablet computer interaction through multimodal input methods involving pen writing and touch manipulation. However, in this work, the natural fluidity of user grip and gestures was limited by the transmission capabilities between the pen and the multi-touch device. Therefore, Matulic et al. [13] installed a downward-facing camera on top of the pen to capture information about pen grip and the surrounding environment.

In the process of 3D spatial interaction, with the increasing complexity of interaction tasks, there is a growing demand for high-precision input. Pham et al. [14] conducted a comparative analysis of the performance of performing pointing tasks in a virtual environment using a mouse, controller, and 3D pen. Key metrics included movement time, error rate, speed, and comfort. The study showed that the 3D pen outperformed VR controllers in these evaluation metrics, and its performance was comparable to that of a mouse. This advantage is mainly attributed to the 3D pen's more efficient utilization of wrist and finger movements for multi-degree-of-freedom operations [12]. Wacker et al. [21] implemented real-time observation of users drawing virtual strokes in the air on a mobile phone using Apple's ARKit technology and visual markers on a 3D-printed pen. However, limitations in depth perception and the absence of physical support in virtual environments make precise drawing challenging. To address this challenge, Elsayed et al. [6] simulated pressure and tactile texture sensations of a pen on a virtual surface through pneumatic feedback and vibration tactile feedback, significantly enhancing users' realistic interaction experience with virtual interfaces.

Currently, research in Human-Pen Interaction predominantly focuses on pen parameters, sensor design, and external physical support, with relatively less attention given to the pen as a tool for perceiving the environment, understanding, and interpreting information. Therefore, this paper proposes a smart pen with active collaborative capabilities, aiming to explore the interaction between the smart pen and teachers during solid geometry teaching.

2.2 Active Human-Computer Collaboration

Human-computer collaboration is not merely an extension of computers performing tasks but involves intelligent systems acquiring, processing vast amounts of participant-related information, predicting participant intent, and proactively identifying issues. This enables machines to possess understanding, learning, and decision-making capabilities, achieving more profound and effective cooperation. In this trend, human-computer collaboration systems exhibit proactive features, requiring systems to perceive, construct, and infer the physical world [18]. Rashed et al. [15] proposed a vision-based museum guide robot system that uses video information from multiple camera sensors to capture visitors' head direction information. It predicts visitors' interests and intentions regarding artworks in real-time, providing visitors with proactive guidance and explanations. Dianatfar et al. [4] argue that communication between humans and robots is not intuitive, fast, or flexible enough. Technologies such as AR, VR, and MR can be applied to real human-computer collaborative scenarios, offering participants a more realistic environment to perceive instructions and environmental conditions.

In our research, to realize active collaborative point-line selection interaction in solid geometry using a smart pen, the first step is to achieve more accurate point-line intention prediction in the solid geometry teaching scenario. Therefore, we particularly focus on the complementary relationship between the teacher's inputted voice information, the smart pen's positional data, and sensor information. By integrating multimodal information, the system becomes better equipped to adapt to the teacher's teaching habits and changes in the teaching environment.

In summary, this paper specifically focuses on the following two research questions:

RQ1: How do we design a smart pen and its teaching assistance system tailored for solid geometry teaching to perceive the teaching interaction needs of teachers?

RQ2: How can active collaboration strategies be utilized to achieve point-line interactions in solid geometry?

3 SYSTEM CONFIGURATION

3.1 Design of the Smart Pen

The smart pen system, PenLab, is an integrated intelligent interaction suite encompassing both hardware and software components. Its hardware comprises a 3D-printed pen body, three touch sensors (inductive coils), and an IMU (Figure 2). Touch sensor TS_1 serves as a function key, TS_2 as the confirm key, and TS_3 functions as the cancel key. As shown in Figure 3, teachers can perform operations such as zooming in, zooming out, and rotating on geometric objects using different keys. The software component of PenLab utilizes the Unity 3D engine and C# for the development of scenes and interactions, building a solid geometry teaching platform that achieves active collaborative interaction between humans and smart pen.

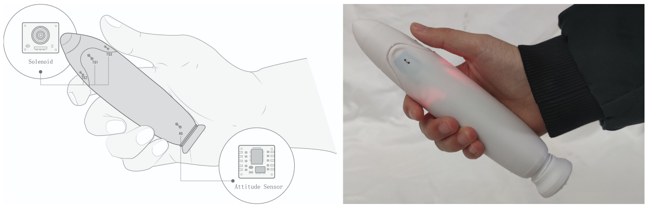

Figure 2: Smart Pen Hardware Structure.

Regarding the spatial tracking of the pen, Li et al. [12] utilized the OptiTrack V120: Trio1, while Wacker et al. [21] employed Apple's ARKit technology and visual markers on a 3D-printed pen. However, these methods present certain limitations in terms of cost and applicability. To balance cost and suitability for teaching, we opted for the Intel RealSense SR300. This device, with a relatively lower cost, offers a depth detection range of 0.2m-2m, meeting the distance requirements for teachers' movements on the podium. We utilized this depth camera to obtain real-time depth information of the smart pen, combining it with YOLOv5 object detection technology to acquire 2D coordinate information, thus obtaining the 3D position information of the smart pen. To improve the real-time and smoothness of the data, we implemented a composite filtering approach, successfully eliminating data jitter and obtaining real-time and smooth position information of the smart pen. Finally, through serial communication, we sent sensor data and position information to the computer end, realizing real-time accurate information feedback between virtual scenes and real scenes (Figure 4).

3.2 Active Collaborative Point-Line Interaction Strategy

3.2.1 Intent Understanding. In the context of solid geometry teaching, the system is influenced by various factors, including environmental noise and teacher-student communication, which may affect the accurate prediction of teacher intentions. Additionally, a single modality may not fully reflect the true intentions, leading to potential understanding ambiguities, for instance, a piece of speech information might correspond to two or more intentions. Furthermore, we observe that multimodal information, including speech data, smart pen position data, and sensor information, often exhibit complementary relationships. For example, when speech involves selecting points to draw auxiliary lines, smart pen position data can provide additional geometric context. This multimodal complementarity enhances the system's intention inference capabilities, contributing to improved performance in complex teaching environments.

For the acquisition of touch sensor information, we establish a tactile perception library. When the teacher long-presses the function key TS_1, the system obtains smart pen position data and speech information. Subsequently, based on the smart pen position data, we calculate the confidence of each node (vertex, line segment), determining the likely selection area of the teacher, i.e., the geometric contextual intent of the smart pen. For speech information, considering the linguistic characteristics of secondary school solid geometry teaching, we construct a speech intent library specific to secondary school solid geometry teaching. We use text similarity calculation based on word2vec [10] to obtain the teacher's speech sub-intent. Next, we establish an intent database, where the intent consists of geometric context sub-intents and speech sub-intents. After obtaining sub-intents from both modalities, meaningful intents are derived by arranging and combining the acquired sub-intents according to the intent database, forming a set of possible intents for participants. To obtain the true intent from the set of possible intents for participants, we conduct weighted fusion based on Dempster's combination rule[19] to obtain the trust allocation values of intents, and the intent with the highest trust allocation value is the teacher's interaction intent. For example, when the teacher long-presses the function key TS_1, touch sensor information is "position information and speech information acquisition," speech input "draw a perpendicular line" obtains the speech sub-intent, at the same time, the system obtains the geometric context

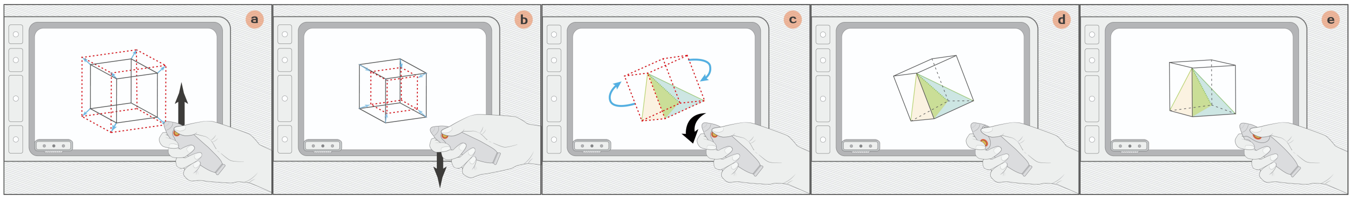

Figure 3: Zoom In, Zoom Out and Rotate: (a) Click the function key TS_1, the cube enlarges following the trend of the smart pen's posture change; (b) Click the function key TS_1, the cube shrinks following the trend of the smart pen's posture change; (c) Short press the function key TS_1, the cube rotates following the posture of the smart pen; (d) Click the confirm key TS_2 to obtain the current size and posture of the cube; (e) Click the cancel key TS_3, the cube returns to its initial size and posture.

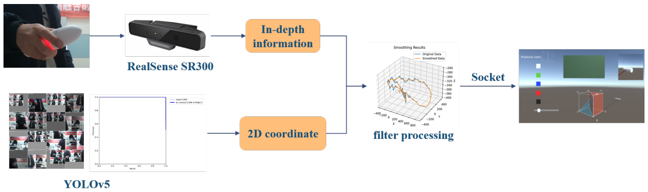

Figure 4: Smart Pen Spatial Position Tracking.

sub-intent based on the smart pen's position information as "draw an auxiliary line from point A to line segment B_1D." Integrating complementary information from both modalities, we derive the teacher's interaction intent as "draw a perpendicular line from point A to edge B_1D."

3.2.2 Active Collaborative Geometry Object Selection. PenLab, upon acquiring the predicted intention, transitions into a stage awaiting teacher actions (Figure 5). Analysis of point selection data from 14 participants revealed an average selection time of 1.69 seconds. Consequently, we set the initial waiting time threshold to 1.69 seconds. During this waiting period, PenLab generates guiding lines to assist teachers in moving the smart pen to the respective position (Figure 1b).

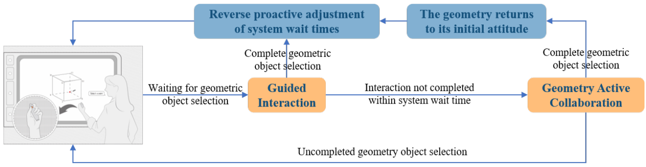

Figure 5: Active Collaborative Point-Line Interaction Strategy.

If the system does not detect the teacher's operation on the point-line intention within the waiting time, the system will actively calculate the rotation angle of the target point or line segment relative to the current geometry's posture, adjust the posture of the geometry, and move the target point or line segment to an area where the teacher can easily operate. To make the system more personalized, the system will adaptively adjust the waiting time. After each waiting period, the system compares the current waiting time (T_{cur}) with the time (t_{user}) the teacher took to complete the operation.

$$T_{cur} = \begin{cases} \alpha * T_{cur}, t_{user} \leq \tau * T_{cur} \\ \beta * T_{cur}, t_{user} > \tau * T_{cur} \end{cases} \tag{1}$$

In this equation, τ represents the threshold for comparing the teacher's operation time with the current waiting time, which we set to 0.8; α represents the scaling factor for values less than 1, used to shorten the waiting time, and we set it to 0.9; β represents the scaling factor for values greater than 1, used to extend the waiting time, and we set it to 1.1. If $t_{user} \leq \tau * T_{cur}$, meaning the teacher completes the point-line selection within the given waiting time, the system will shorten the next waiting time. If $t_{user} > \tau * T_{cur}$, indicating the teacher needs more time to operate, the system will extend the next waiting time.

When the teacher successfully completes the point-line interaction, the target point or line segment will turn red, indicating the successful selection by the teacher. The geometry's pose returns to the initial state, executing the corresponding intention. The system proactively adjusts the waiting time, preparing for the next interaction. If the teacher does not succeed in the selection, the geometric body's pose returns to the initial state, and the system re-predicts the teacher's intention. For example, if the system's predicted intent is to "select point C," it will wait for the participant to interact while generating a guiding line between the smart pen and point C, facilitating accurate selection of point C by the participant. If the participant fails to select point C correctly within the system's waiting time, the geometry will adjust its posture autonomously, moving point C to an area that is easier for the participant to operate.

4 EVALUATION

We invited 14 participants with teaching qualifications (7 males and 7 females; M=24.4 years; STD=1.04 years). The experiment took place within the field of view of the depth camera, simulating teachers performing solid geometry teaching operations (Figure 1). Before the experiment began, we provided participants with instruction on using PenLab and GeoGebra [22], as well as on solving for the volume of a triangular pyramid, to ensure they had a certain level of understanding and familiarity with both tools. Furthermore, solving for the volume of a triangular pyramid, as a typical geometric problem, is key to understanding and mastering the basic concepts of solid geometry. The geometric principles and calculation methods involved are complex and challenging enough to ensure that participants can engage in the experiment using their

Table 1: Interaction Intentions

Intention Number	Intentions
1	Select point C
2	Select edge CC_1
3	Select edge CD
4	Connect points B and D
5	Connect points B_1 and D with a dashed line
6	Draw a perpendicular line from point A to line segment B_1D
7	Draw a perpendicular line from point B_1 to line segment AD_1

mathematical background and teaching skills without it being too simple to fully demonstrate the features and effectiveness of the PenLab and GeoGebra tools. Therefore, we used solving for the volume of a triangular pyramid as an experimental example and required participants to apply similar geometric principles and calculation methods to solve this problem, ensuring a fair comparison and evaluation of PenLab and GeoGebra. During the experiment, to fully simulate the teaching environment, we did not require absolute silence. Participants could rest at any time to alleviate fatigue and were encouraged to seek help or ask questions at any time to address any difficulties or doubts.

Firstly, participants expressed their interaction intentions for specific tasks (Table 1). Subsequently, participants performed experiments to solve the volume of a triangular pyramid using GeoGebra and PenLab, followed by completing a NASA-TLX survey [2] and a SUS survey [7]. We recorded the system's accuracy in predicting participants' interaction intentions, the number of successful selections of geometric objects, average interaction time, and participants' evaluations of GeoGebra and PenLab.

5 RESULT AND DISCUSSION

Our research results clearly indicate that PenLab can capture and understand teachers' interactive behaviors in real-time and assist teachers in achieving precise point-line interactions through proactive guidance and adjustment of geometric poses.

In the 98 interactive intentions of 14 participants, the system correctly predicted 94 intentions, achieving an accuracy rate of 95.91%. This indicates that through the complementary nature of multimodal information, the system can understand teachers' interactive intentions relatively accurately. Regarding the selection of geometric objects, Table 2 shows that the success rate of geometric object selection using only guided interaction is 85.71%. For geometric objects that were not successfully selected within the system's waiting time, the system actively cooperates by adjusting geometric poses, increasing the success rate of geometric object selection to 97.61%. In addition, as shown in Figure 6, the total average interaction time for this strategy is 1.34s. Compared to the participants' initial point selection time of 1.69s, this strategy ensures the successful selection of geometric objects while increasing the average interaction time by 0.35s. Moreover, in the guided + cooperative interaction, the average interaction time for participants is relatively high. This is because if participants do not complete geometric object selection within the system's waiting time, the system actively cooperates to improve the success rate of geometric object selection, but at the same time, this inevitably increases the interaction time.

Table 2: Geometric Object Selection Success Rate

	Guided Interaction	Guided + Cooperative Interaction
Successful Selections	36	41
Total Selections	42	42
Selection Success Rate	85.71%	97.61%

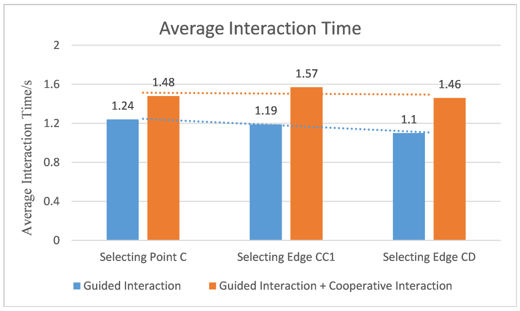

Figure 6: Average Interaction Time.

In the NASA-TLX questionnaire, participants rated six factors influencing task workload: Mental Demand (MD), Physical Demand (PhD), Temporal Demand (TD), Performance Level (Per), Effort (E), and Frustration Level (FL). Among these, MD refers to the cognitive effort required during task completion, while PhD quantifies the physical exertion involved. TD assesses whether participants perceive time pressure during tasks. Per measures satisfaction with personal task performance, while E quantifies the exertion required to achieve self-assessed performance levels. FL represents post-task frustration. We calculated the correlation, mean, and mean square deviation for each factor in the 28 questionnaires, where the correlation is proportional to the width of each factor in Figure 7. As shown in Figure 7, MD, Per, and E have a greater correlation with the task, indicating that participants are more concerned about the performance of PenLab and GeoGebra in the teaching process. In terms of PhD and TD, PenLab is lower than GeoGebra, indicating that the geometric object selection strategy reduces participants' physical load and interaction time, making PenLab more relaxed than GeoGebra. Through weighted calculation of these six factors, PenLab's total workload value is 5.31, while GeoGebra's total workload value is 5.68, further confirming the advantages of using PenLab for solid geometry teaching.

The SUS questionnaire (A.1) consists of 5 questions, focusing on (1) participants' acceptance of using PenLab for solid geometry teaching (Q1 and Q2), (2) participants' preference for three geometric teaching methods (Q3 and Q4), and (3) participants' confidence in PenLab teaching (Q5). Participants scored based on the survey rating criteria (A.2), and the subjective feedback results are shown in Figure 8. All participants expressed willingness to use PenLab for solid geometry teaching, with nearly 90% acknowledging the positive impact of PenLab in facilitating teaching. Approximately 80% of participants favored the use of PenLab over traditional chalkboard teaching and GeoGebra. Additionally, nearly 90% of participants perceived teaching with PenLab as more comfortable and confidence-inspiring. Overall, participants' evaluation of PenLab

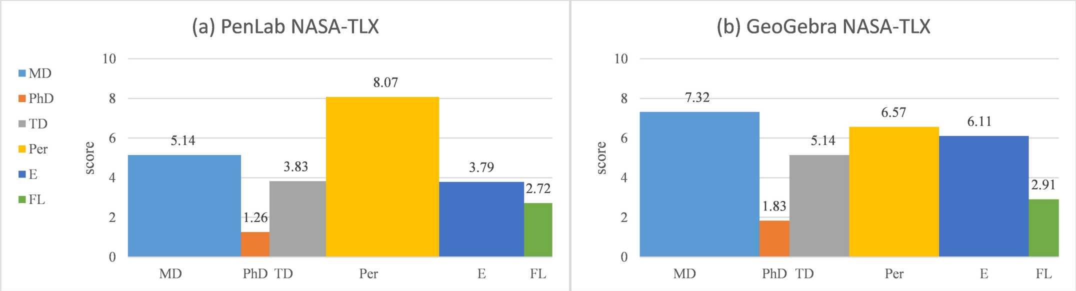

Figure 7: (a) PenLab NASA-TLX; (b) GeoGebra NASA-TLX.

is generally positive, recognizing its acceptance and superiority in solid geometry teaching.

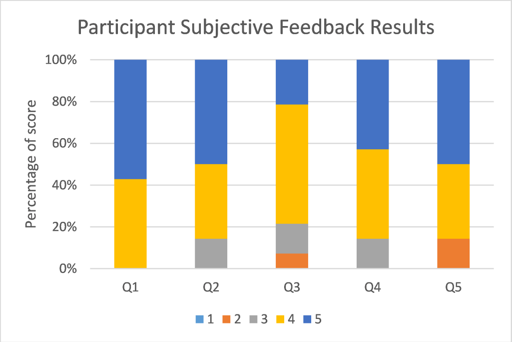

Figure 8: Participant Subjective Feedback Results.

6 CONCLUSION AND FUTURE WORK

This paper introduces a smart pen system, PenLab, consisting of an active cooperative smart pen designed for solid geometry teaching and a virtual geometry teaching platform. The system can perceive the teaching interaction needs of teachers and assist them in achieving precise point-line interaction in solid geometry using an active cooperative strategy. The research results indicate that our PenLab is better suited for solid geometry teaching scenarios, providing more precise and flexible interaction methods to assist geometry teaching.

However, this study still has some limitations. Some participants suggested that the design of the smart pen could be more compact for ease of grip. In the next phase, we plan to design an smart pen that better aligns with teachers' usage habits, including optimizing its size and button placement. Furthermore, future research could explore how to expand PenLab to meet teaching requirements in different fields. Although this study focuses on solid geometry teaching, similar proactive collaborative methods may have potential applications in other disciplines and domains.

Overall, this study provides novel technological support for teachers in the teaching of solid geometry and offers valuable references for exploring the application of pen-based interaction technology in 3D space. We anticipate that the outcomes of this research will have a positive impact in the field of educational technology, contributing to the improvement of teaching effectiveness.

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A APPENDICES

A.1 PenLab SUS Questionnaire

Question	
Q1	I am willing to use PenLab for solid geometry teaching.
Q2	I am very interested in PenLab.
Q3	I prefer PenLab over traditional chalkboard teaching.
Q4	I prefer PenLab over GeoGebra.
Q5	I feel confident in using PenLab for teaching.

A.2 Survey Score Criteria

Question	1	2	3	4	5
Q1	no	maybe no	neutral	maybe yes	yes
Q2	no	maybe no	neutral	maybe yes	yes
Q3	dislike	a little dislike	neutral	like	very like
Q4	dislike	a little dislike	neutral	like	very like
Q5	no	maybe no	neutral	maybe yes	yes